Hash Function *Luffa*

Specification Ver. 2.0.1

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Contents

1 Introduction

This document specifies a family of cryptographic hash function algorithms *Luffa*. The input and the output lengths of the algorithms are summarized in Table 1.

Table 1: Input and output lengths

Algorithm	Message length (bits)	Hash length (bits)	Security (bits)
Luffa-224	$< 2^{64}$	224	112
Luffa-256	$< 2^{64}$	256	128
Luffa-384	$< 2^{128}$	384	192
Luffa-512	$< 2^{128}$	512	256

Firstly, the notations used in the document are defined in Section 2. The hash function *Luffa* consists of the chaining and the mixing function used in each round of the chaining. The chaining and the underlying mixing function are described in Section 3 and 4 respectively. An optional usage of the hash function *Luffa* is given in Section 5. In addition, some useful information to implement the hash function such as the test vectors is given in Appendices.

2 Preliminary

In this section, the basic terms and notations to describe the specification of *Luffa* are defined.

2.1 Notations

2.1.1 Parameters

L:	The message length in bits
L':	The padded message length in bits
N:	The number of message block (of 256 bits)
w:	The number of sub-permutations (described in 3.2)
n_h:	The hash length
n_b:	The block length (Fixed to 256 bits in this document)
V_j:	The starting variables
$H_j^{(i)}$:	The variable which specifies the intermediate values of the state at i-th round, j-th block
$M^{(i)}$:	The message block at the i-th round
i:	A subscript which specifies the round
j:	A subscript which specifies the sub-permutation
k:	A subscript which specifies the word
l:	A subscript which specifies the bit position in a word
r:	A subscript which specifies the step
MI:	The message injection function
P:	The permutation of $n_b w$ bits
Q_j:	The permutation dealing with j-th block of n_b bits
OF:	The output function
$b_{j,k,l}$:	The variable which specifies the k-th word, l-th bit of the input of the j-th block permutation Q_j
$a_{j,k,l}^{(i,r)}$:	The variable which specifies the k-th word, l-th bit of the input of i-th round, j-th block, r-th step function
$x_{j,k,l}^{(i,r)}$:	The variable which specifies the k-th word, l-th bit of the output of `SubCrumb` at i-th round, j-th block, r-th step
$y_{j,k,l}^{(i,r)}$:	The variable which specifies the k-th word, l-th bit of the output of `MixWord` at i-th round, j-th block, r-th step

$c^{(r)}_{j,k,l}$: The variable which specifies the k-th word, l-th bit of the constant used in j-th block, r-th step function

2.1.2 Symbols

In this paper, the following symbols are used to identify the operations.

$\oplus$	Bitwise XOR operation
$\wedge$	Bitwise AND operation
$\|\|$	Concatenation of two bit strings
$\ggg n$	Rotation n bits to the right (A 32-bit register is expected)
$\lll n$	Rotation n bits to the left (A 32-bit register is expected)
`0x`	Hexadecimal prefix

On the other hand, some pseudo codes are given in the paper. They are written in C language manner and 32-bit registers are expected. In order to remove any ambiguity, we also list up the operation used in the pseudo codes as follows:

`^`	XOR operation
`|`	OR operation
`>>` n	Shift n bits to the right
`<<` n	Shift n bits to the left

2.2 Data Structure

The basic data size is a 32-bit and it is called a *word*. A 4 bytes data is stored to a word in the big endian manner. In other words, the given 4 bytes data $x_0, \ldots, x_3$ is stored into a word a as follows:

$$a = [\mathrm{MSB}] \quad x_0||x_1||x_2||x_3 \quad [\mathrm{LSB}],$$

where [MSB] (and [LSB]) means the most (and least) significant byte of the word.

In the specification of *Luffa*, a 256-bit data block is stored in 8 32-bit registers. In order to remove any ambiguity, we also define the ordering of a

32 bytes data in 8 words. A 32 bytes data $X = x_0, x_1, \ldots, x_{31}$ is stored to 8 32-bit registers $a_0, \ldots, a_7$ in the following manner:

$$\begin{aligned} X &= [\text{MSW}] \quad a_0||a_1||\cdots||a_7 \quad [\text{LSW}], \\ a_k &= [\text{MSB}] \quad x_{4k}||x_{4k+1}||x_{4k+2}||x_{4k+3} \quad [\text{LSB}], \quad 0 \leq k < 8, \end{aligned}$$

where [MSW] (and [LSW]) means the most (and least) significant word.

A bit position in a word sequence is denoted by subscripts. Let $a_0, \ldots, a_n$ be a word sequence. Then the l-th bit (from the least significant bit) of the k-th word is denoted by $a_{k,l}$, where the least significant bit is the 0-th bit. In other words, the bit information of a_k is given by

$$a_k = [\text{msb}] \quad a_{k,31}||a_{k,30}||\cdots||a_{k,1}||a_{k,0} \quad [\text{lsb}],$$

where [msb] and [lsb] mean the most and the least significant bit of the word respectively.

2.3 Iterations

The message processing of *Luffa* is a chaining of a mixing function of a fixed length input and a fixed length output. We call the mixing function as a *round function*. The outline of the mixing function is defined in Section 3. A term *round* means the procedure to apply the round function.

The building block of the round function is a family of non-linear permutations defined in Section 4. It consists of iterations of a sub-function called a *step function*. A term *step* means the procedure to apply the step function.

In order to clarify the round, the super-script with a parenthesis is used. I.e., the input to the i-th round function is denoted by $X^{(i-1)}$. The corresponding output of the round function is denoted by $X^{(i)} = \texttt{Round}(X^{(i-1)})$. In the same manner, the input to the r-th step function of the i-th round is denoted by $X^{(i-1,r-1)}$. The corresponding output of the step function is denoted by $X^{(i-1,r)} = \texttt{Step}(X^{(i-1,r-1)})$. The round can be abbreviated if it is not necessary in the context.

The intermediate state of *Luffa* consists of $8w$ words, where $w \geq 3$ is a positive integer (See Table 2 for the choice of w). An 8 word data is called a *block*. The l-th bit of the input of i-th round, r-th step, j-th block, k-th word is denoted by $a^{(i-1,r-1)}_{j,k,l}$.

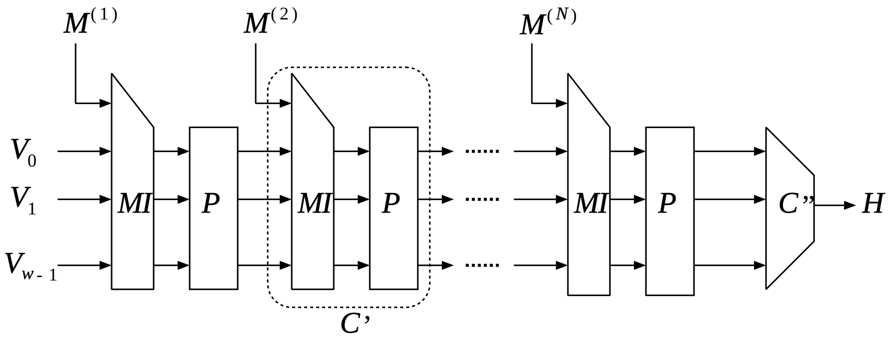

Figure 1: A generic construction of a hash function based on a permutation

3 Chaining

The chaining of *Luffa* is a variant of a sponge function [1, 2]. Figure 1 shows the basic structure of the chaining. The chaining of a hash function consists of the intermediate mixing C' (called a round function) and the finalization C''. In addition to above two functions, the message padding is defined in this section. The starting variables $V_0, V_1, \ldots, V_{w-1}$ used in the chaining are given in Appendix A.

3.1 Message Padding

Suppose that the length of the message M is l bits. First of all, the bit string $100\ldots0$ is appended to the end of the message. The number of zeros k should be the smallest non-negative integer which satisfies the equation $l + 1 + k \equiv 0 \bmod 256$. Therefore the length of the padded message should be a multiple of 256 bits.

3.2 Round Function

The round function is a composition of a message injection function MI and a permutation P of $w \cdot n_b$ bits input. The permutation is divided into plural sub-permutation Q_j of n_b bits input (See Figure 2). Let the input of the i-th round be $(H_0^{(i-1)}, \ldots, H_{w-1}^{(i-1)})$, then the output of the i-th round is given by

$$\begin{aligned} H_j^{(i)} &= Q_j(X_j), \quad 0 \leq j < w, \\ X_0 || \cdots || X_{w-1} &= MI(H_0^{(i-1)}, \ldots, H_{w-1}^{(i-1)}, M^{(i)}), \end{aligned}$$

where $H_j^{(0)} = V_j$.

In the specification of *Luffa*, the input length of the sub-permutation Q_j is fixed to $n_b = 256$ bits, and the number of the sub-permutations w is defined in Table 2.

Table 2: The width of the registers

Hash length n_h	Number of permutations w
224	3
256	3
384	4
512	5

The message injection functions can be represented by the matrix over a ring $\mathrm{GF}(2^8)^{32}$. The definition polynomial of the field is given by $\phi(x) = x^8 + x^4 + x^3 + x + 1$. The map from an 8 word value $(a_0, \ldots, a_7)$ to an element of the ring is defined by $(\sum_{0 \leq k < 8} a_{k,l} x^k)_{0 \leq l < 32}$. Note that the least significant word a_7 is the coefficient of the heading term x^7 in the polynomial representation. In order to remove any ambiguity, we also define the multiplication by `0x02` (equivalent to x in the polynomial representation) as the following pseudo code:

```
tmp  = a[7];
a[7] = a[6];
a[6] = a[5];
a[5] = a[4];
a[4] = a[3] ^ tmp;
a[3] = a[2] ^ tmp;
a[2] = a[1];
a[1] = a[0] ^ tmp;
a[0] = tmp;
```

In the following, the matrices representing the message injection functions MI for $w = 3, 4, 5$ are defined. The way of implementing MI only with XORings and multiplications by `0x02` is shown in Appendix E.

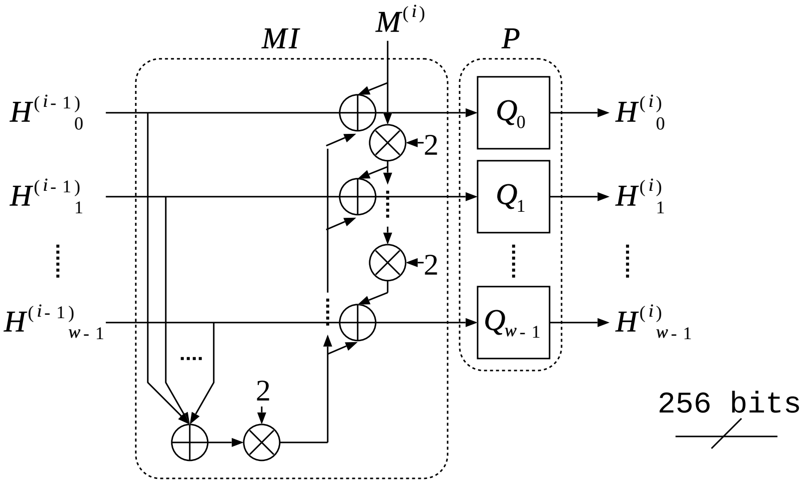

Figure 2: The round function (The message injection function is for $w = 3$)

3.2.1 Message Injection Function for $w = 3$

The matrix representation of the message injection function MI for $w = 3$ is defined by

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} 3 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} H_0^{(i-1)} \\ H_1^{(i-1)} \\ H_2^{(i-1)} \\ M^{(i)} \end{pmatrix},$$

where numerics `0x01`, `0x02`, `0x03`, `0x04` correspond to polynomials 1, x, $x+1$, x^2 respectively. The prefix `0x` is omitted in order to reduce the redundancy.

3.2.2 Message Injection Function for $w = 4$

The matrix representation of the message injection function MI for $w = 4$ is defined by

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \end{pmatrix} = \begin{pmatrix} \mathtt{4} & \mathtt{6} & \mathtt{6} & \mathtt{7} & \mathtt{1} \\ \mathtt{7} & \mathtt{4} & \mathtt{6} & \mathtt{6} & \mathtt{2} \\ \mathtt{6} & \mathtt{7} & \mathtt{4} & \mathtt{6} & \mathtt{4} \\ \mathtt{6} & \mathtt{6} & \mathtt{7} & \mathtt{4} & \mathtt{8} \end{pmatrix} \begin{pmatrix} H_0^{(i-1)} \\ H_1^{(i-1)} \\ H_2^{(i-1)} \\ H_3^{(i-1)} \\ M^{(i)} \end{pmatrix}.$$

3.2.3 Message Injection Function for $w = 5$

The matrix representation of the message injection function MI for $w = 5$ is defined by

$$\begin{pmatrix} X_0 \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{pmatrix} = \begin{pmatrix} \mathtt{0F} & \mathtt{08} & \mathtt{0A} & \mathtt{0A} & \mathtt{08} & \mathtt{01} \\ \mathtt{08} & \mathtt{0F} & \mathtt{08} & \mathtt{0A} & \mathtt{0A} & \mathtt{02} \\ \mathtt{0A} & \mathtt{08} & \mathtt{0F} & \mathtt{08} & \mathtt{0A} & \mathtt{04} \\ \mathtt{0A} & \mathtt{0A} & \mathtt{08} & \mathtt{0F} & \mathtt{08} & \mathtt{08} \\ \mathtt{08} & \mathtt{0A} & \mathtt{0A} & \mathtt{08} & \mathtt{0F} & \mathtt{10} \end{pmatrix} \begin{pmatrix} H_0^{(i-1)} \\ H_1^{(i-1)} \\ H_2^{(i-1)} \\ H_3^{(i-1)} \\ H_4^{(i-1)} \\ M^{(i)} \end{pmatrix}.$$

3.3 Finalization

The finalization consists of iterations of an output function OF and a round function with a fixed message `0x00...0`. A blank round with a fixed message block `0x00...0` is applied at the beginning of the finalization.

The output function OF XORs all block values and outputs the resultant 256-bit value. Let the output at the i-th iteration be Z_i, then the output function is defined by

$$Z_i = \bigoplus_{j=0}^{w-1} H_j^{(N+i+1)}.$$

The detailed output words are defined in Table 3. In fact, *Luffa*-224 just truncates the last one word of the output of *Luffa*-256.

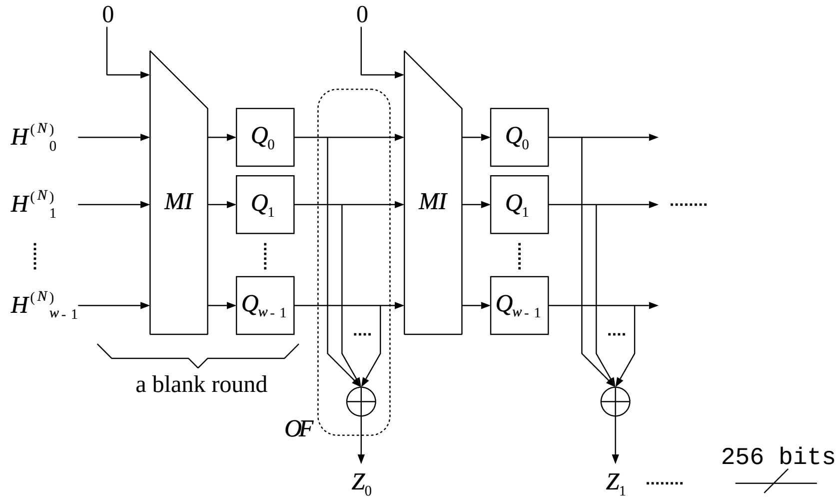

Figure 3: The finalization function

Table 3: The hash values

Hash length n_h	Hash value H
224	$Z_{0,0}\|\|\cdots\|\|Z_{0,6}$
256	$Z_{0,0}\|\|\cdots\|\|Z_{0,7}$
384	$Z_{0,0}\|\|\cdots\|\|Z_{0,7}\|\|Z_{1,0}\|\|\cdots\|\|Z_{1,3}$
512	$Z_{0,0}\|\|\cdots\|\|Z_{0,7}\|\|Z_{1,0}\|\|\cdots\|\|Z_{1,7}$

4 Non-Linear Permutation

In this section, the detailed specification of the permutation Q_j is given. Some subscripts such as i, j, r will be omitted in this section if it is clear in the context. For example, $a^{(i,r)}_{j,k,l}$ is denoted by $a_{k,l}$.

4.1 Outline

The *Luffa* hash function uses a non-linear permutation Q_j whose input and output length is 256 bits. The permutation Q_j is defined as a composition of an input tweak and iterations of a step function `Step`. The number of iterations of a step function is 8 and the tweak is applied only once per a

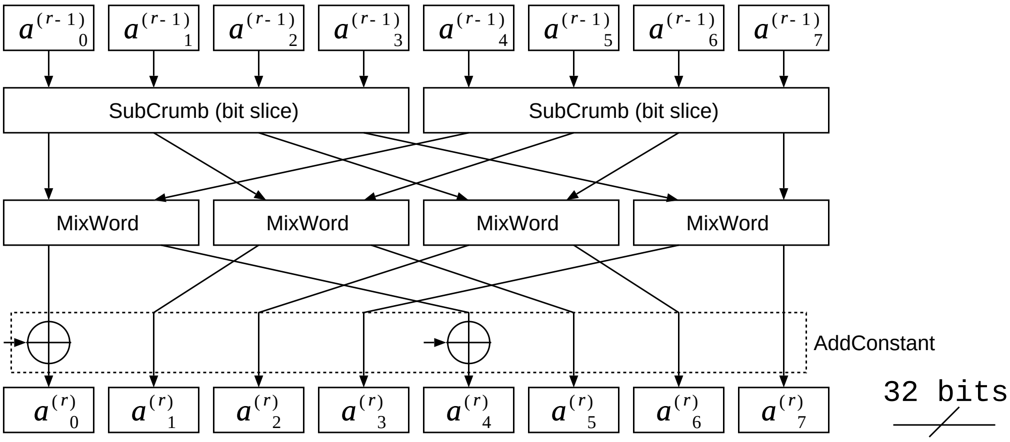

Figure 4: The step function

permutation.

At the beginning of the step function process, the 256 bits data stored in 8 32-bit registers is denoted by $a_k^{(r)}$ for $0 \leq k < 8$. The data before applying the permutation Q_j is denoted by b_k and the data after the tweak is denoted by $a_k^{(0)}$. The step function consists of the following three functions; `SubCrumb`, `MixWord`, `AddConstant`. The pseudo code for Q_j is given by

```
Permute(a[8], j){ //Permutation Q_j
    Tweak(a);
    for (r = 0; r < 8; r++){
        SubCrumb(a[0],a[1],a[2],a[3]);
        SubCrumb(a[5],a[6],a[7],a[4]);
        for (k = 0; k < 4; k++)
            MixWord(a[k],a[k+4]);
        AddConstant(a, j, r);
    }
}
```

Each function is described below in turn and the tweaks are described in Section 4.5.

4.2 `SubCrumb`

`SubCrumb` substitutes l-th bits of a_0, a_1, a_2, a_3 (or a_4, a_5, a_6, a_7) by an Sbox S defined by

$$S[16] = \{\texttt{13}, \texttt{14}, \texttt{0}, \texttt{1}, \texttt{5}, \texttt{10}, \texttt{7}, \texttt{6}, \texttt{11}, \texttt{3}, \texttt{9}, \texttt{12}, \texttt{15}, \texttt{8}, \texttt{2}, \texttt{4}\}.$$

Let the output of `SubCrumb` be x_0, x_1, x_2, x_3 (or x_4, x_5, x_6, x_7). Then the substitution by `SubCrumb` is given by

$$\begin{aligned} x_{3,l}||x_{2,l}||x_{1,l}||x_{0,l} &= S[a_{3,l}||a_{2,l}||a_{1,l}||a_{0,l}], \quad 0 \leq l < 32, \\ x_{4,l}||x_{7,l}||x_{6,l}||x_{5,l} &= S[a_{4,l}||a_{7,l}||a_{6,l}||a_{5,l}], \quad 0 \leq l < 32. \end{aligned}$$

Note that the latter four words a_4, a_5, a_6, a_7 are input to the Sboxes in different order from the first four words.

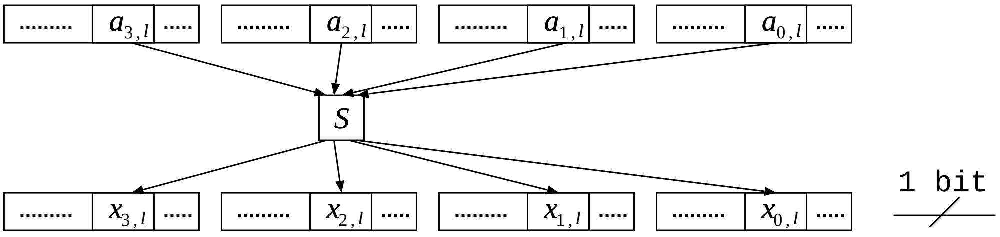

Figure 5: The input and output bits of the Sbox

Appendix D shows the optimal instruction set for Intel Core2 Duo processors [1].

4.3 `MixWord`

`MixWord` is a linear permutation of two words. Figure 6 shows the outline of `MixWord`. Let the output words be y_k and y_{k+4} where $0 \leq k < 4$. Then `MixWord` is given by the following equations:

$$y_{k+4} = x_{k+4} \oplus x_k,$$

---

[1] Intel is a registered trademark and Core is the name of products of Intel Corporation in the U.S. and other countries.

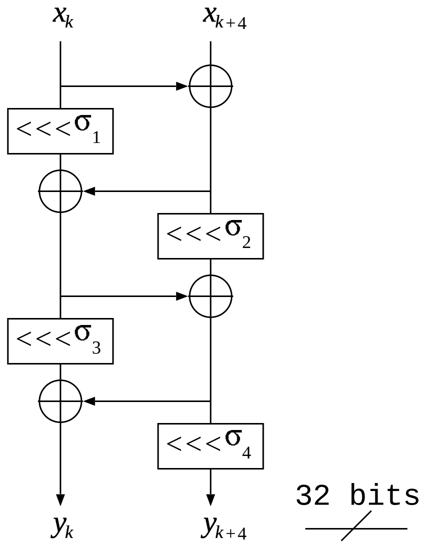

Figure 6: `MixWord`

$$\begin{aligned}
y_k &= x_k \lll \sigma_1, \\
y_k &= y_k \oplus y_{k+4}, \\
y_{k+4} &= y_{k+4} \lll \sigma_2, \\
y_{k+4} &= y_{k+4} \oplus y_k, \\
y_k &= y_k \lll \sigma_3, \\
y_k &= y_k \oplus y_{k+4}, \\
y_{k+4} &= y_{k+4} \lll \sigma_4.
\end{aligned}$$

The parameters σ_i are given by $\sigma_1 = 2, \sigma_2 = 14, \sigma_3 = 10, \sigma_4 = 1$.

4.4 `AddConstant`

`AddConstant` is given by

$$a_{j,k}^{(r)} = y_{j,k}^{(r-1)} \oplus c_{j,k}^{(r-1)}, \quad k = 0, 4.$$

Note that the step constant $c_{j,k}^{(r-1)}$ is not equal to $c_{j',k}^{(r-1)}$ if $j \neq j'$. The step constants are generated sequentially from fixed initial values $c_{j,L}^{(0)}$ and $c_{j,R}^{(0)}$.

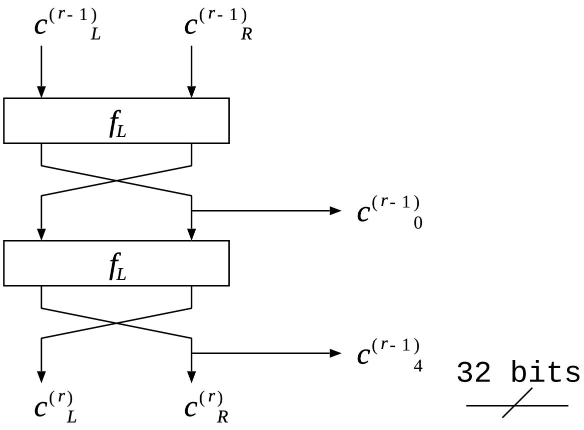

Figure 7: Constant generator

The initial values are given in Appendix B. The constant generation function generates two 32-bit constants $c^{(r-1)}_{j,0}$ and $c^{(r-1)}_{j,4}$ in the following manner:

$$\begin{aligned} t_L\|t_R &= c^{(r-1)}_{j,L}\|c^{(r-1)}_{j,R}, \\ t_L\|t_R &= f_L(t_L\|t_R), \\ c^{(r-1)}_{j,0} &= t_L, \\ t_L\|t_R &= f_L(t_R\|t_L), \\ c^{(r-1)}_{j,4} &= t_L, \\ c^{(r)}_{j,L}\|c^{(r)}_{j,R} &= t_R\|t_L, \end{aligned}$$

where the function f_L is an LFSR of Galois configuration with defined by the polynomial g given by

$$\begin{aligned} g(x) = {} & x^{64}+x^{63}+x^{62}+x^{58}+x^{55}+x^{54}+x^{52}+x^{50}+x^{49}+x^{46}+x^{43} \\ & +x^{40}+x^{38}+x^{37}+x^{35}+x^{34}+x^{30}+x^{28}+x^{26}+x^{24}+x^{23}+x^{22} \\ & +x^{18}+x^{17}+x^{12}+x^{11}+x^{10}+x^{7}+x^{3}+x^{2}+1. \end{aligned}$$

In order to remove any ambiguity, we also define a step of the constant generator as the following pseudo code:

```
c = tl >> 31;
```

```
tl = (tl << 1) | (tr >> 31);
tr = tr << 1;
if (c == 1){ tl ^= 0xc4d6496c; tr ^= 0x55c61c8d; }
SWAP(tl, tr);
step_const[j][r][k] = tr; /* k=0,4 */
```

4.5 Tweaks

For each permutation Q_j, the least significant four words of a 256-bit input are rotated j bits to the left in 32-bit registers. Let the j-th block, k-th word input be $b_{j,k}$ and the tweaked word (namely the input to the first step function) be $a^{(0)}_{j,k}$, then the tweak is defined by

$$\begin{aligned} a^{(0)}_{j,k,l} &= b_{j,k,l}, \quad 0 \le k < 4, \\ a^{(0)}_{j,k,l} &= b_{j,k,(l-j \bmod 32)}, \quad 4 \le k < 8. \end{aligned}$$

5 Optional Usage

Dispite of the size of the outputs being specified in Section 3.3, the design of *Luffa* allows to generate bit strings of arbitrary length by iterating the output function OF and the round function `Round`. This feature is useful for some applications. On the other hand, it should be pointed out that a longer output with a small w does not improve the security level.

References

[1] G. Bertoni, J. Daemen, M. Peeters and G. Van Assche, "Sponge Functions," Ecrypt Hash Workshop 2007.

[2] G. Bertoni, J. Daemen, M. Peeters and G. Van Assche, "On the Indifferentiability of the Sponge Construction," *Advances in Cryptology, Eurocrypt 2008*, pp. 181–197, 2008.

[3] National Institute of Standards and Technology, "Secure Hash Standard," FIPS 180-2.

[4] National Institute of Standards and Technology, “Digital Sigunature Standard,” FIPS 186-2.

[5] National Institute of Standards and Technology, “The Keyed-Hash Message Authentication Code (HMAC),” FIPS 198.

[6] National Institute of Standards and Technology, “Recommendation for Pair-Wise Key Establishment Schemes Using Discrete Logarithm Cryptography,” SP 800-56A.

[7] National Institute of Standards and Technology, “Recommendation for Number Generation Using Deterministic Random Bit Generators (DRBGs),” SP 800-90.

[8] National Institute of Standards and Technology, “The Advanced Encryption Standard Algorithm Validation Suite (AESAVS)”.

A Starting Variables

The values are taken from [8] Appendix C.1.

$$V_{0,0} = \texttt{0x6d251e69}, V_{0,1} = \texttt{0x44b051e0}, V_{0,2} = \texttt{0x4eaa6fb4}, V_{0,3} = \texttt{0xdbf78465},$$
$$V_{0,4} = \texttt{0x6e292011}, V_{0,5} = \texttt{0x90152df4}, V_{0,6} = \texttt{0xee058139}, V_{0,7} = \texttt{0xdef610bb},$$

$$V_{1,0} = \texttt{0xc3b44b95}, V_{1,1} = \texttt{0xd9d2f256}, V_{1,2} = \texttt{0x70eee9a0}, V_{1,3} = \texttt{0xde099fa3},$$
$$V_{1,4} = \texttt{0x5d9b0557}, V_{1,5} = \texttt{0x8fc944b3}, V_{1,6} = \texttt{0xcf1ccf0e}, V_{1,7} = \texttt{0x746cd581},$$

$$V_{2,0} = \texttt{0xf7efc89d}, V_{2,1} = \texttt{0x5dba5781}, V_{2,2} = \texttt{0x04016ce5}, V_{2,3} = \texttt{0xad659c05},$$
$$V_{2,4} = \texttt{0x0306194f}, V_{2,5} = \texttt{0x666d1836}, V_{2,6} = \texttt{0x24aa230a}, V_{2,7} = \texttt{0x8b264ae7},$$

$$V_{3,0} = \texttt{0x858075d5}, V_{3,1} = \texttt{0x36d79cce}, V_{3,2} = \texttt{0xe571f7d7}, V_{3,3} = \texttt{0x204b1f67},$$
$$V_{3,4} = \texttt{0x35870c6a}, V_{3,5} = \texttt{0x57e9e923}, V_{3,6} = \texttt{0x14bcb808}, V_{3,7} = \texttt{0x7cde72ce},$$

$$V_{4,0} = \texttt{0x6c68e9be}, V_{4,1} = \texttt{0x5ec41e22}, V_{4,2} = \texttt{0xc825b7c7}, V_{4,3} = \texttt{0xaffb4363},$$
$$V_{4,4} = \texttt{0xf5df3999}, V_{4,5} = \texttt{0x0fc688f1}, V_{4,6} = \texttt{0xb07224cc}, V_{4,7} = \texttt{0x03e86cea}.$$

B Constants

B–1 Initial Values

The initial values of the constant generator for Q_j are taken from [8] Appendix C.2.

$$\begin{aligned}
c^{(0)}_{0,L} &= \texttt{0x181cca53}, & c^{(0)}_{0,R} &= \texttt{0x380cde06},\\
c^{(0)}_{1,L} &= \texttt{0x5b6f0876}, & c^{(0)}_{1,R} &= \texttt{0xf16f8594},\\
c^{(0)}_{2,L} &= \texttt{0x7e106ce9}, & c^{(0)}_{2,R} &= \texttt{0x38979cb0},\\
c^{(0)}_{3,L} &= \texttt{0xbb62f364}, & c^{(0)}_{3,R} &= \texttt{0x92e93c29},\\
c^{(0)}_{4,L} &= \texttt{0x9a025047}, & c^{(0)}_{4,R} &= \texttt{0xcff2a940}.
\end{aligned}$$

B–2 $w = 3$

$$c^{(0)}_{0,0} = \texttt{0x303994a6}, \qquad c^{(0)}_{0,4} = \texttt{0xe0337818}$$
$$c^{(1)}_{0,0} = \texttt{0xc0e65299}, \qquad c^{(1)}_{0,4} = \texttt{0x441ba90d}$$
$$c^{(2)}_{0,0} = \texttt{0x6cc33a12}, \qquad c^{(2)}_{0,4} = \texttt{0x7f34d442}$$
$$c^{(3)}_{0,0} = \texttt{0xdc56983e}, \qquad c^{(3)}_{0,4} = \texttt{0x9389217f}$$
$$c^{(4)}_{0,0} = \texttt{0x1e00108f}, \qquad c^{(4)}_{0,4} = \texttt{0xe5a8bce6}$$
$$c^{(5)}_{0,0} = \texttt{0x7800423d}, \qquad c^{(5)}_{0,4} = \texttt{0x5274baf4}$$
$$c^{(6)}_{0,0} = \texttt{0x8f5b7882}, \qquad c^{(6)}_{0,4} = \texttt{0x26889ba7}$$
$$c^{(7)}_{0,0} = \texttt{0x96e1db12}, \qquad c^{(7)}_{0,4} = \texttt{0x9a226e9d}$$

$$c^{(0)}_{1,0} = \texttt{0xb6de10ed}, \qquad c^{(0)}_{1,4} = \texttt{0x01685f3d}$$
$$c^{(1)}_{1,0} = \texttt{0x70f47aae}, \qquad c^{(1)}_{1,4} = \texttt{0x05a17cf4}$$
$$c^{(2)}_{1,0} = \texttt{0x0707a3d4}, \qquad c^{(2)}_{1,4} = \texttt{0xbd09caca}$$
$$c^{(3)}_{1,0} = \texttt{0x1c1e8f51}, \qquad c^{(3)}_{1,4} = \texttt{0xf4272b28}$$
$$c^{(4)}_{1,0} = \texttt{0x707a3d45}, \qquad c^{(4)}_{1,4} = \texttt{0x144ae5cc}$$
$$c^{(5)}_{1,0} = \texttt{0xaeb28562}, \qquad c^{(5)}_{1,4} = \texttt{0xfaa7ae2b}$$
$$c^{(6)}_{1,0} = \texttt{0xbaca1589}, \qquad c^{(6)}_{1,4} = \texttt{0x2e48f1c1}$$
$$c^{(7)}_{1,0} = \texttt{0x40a46f3e}, \qquad c^{(7)}_{1,4} = \texttt{0xb923c704}$$

$$c^{(0)}_{2,0} = \texttt{0xfc20d9d2}, \qquad c^{(0)}_{2,4} = \texttt{0xe25e72c1}$$
$$c^{(1)}_{2,0} = \texttt{0x34552e25}, \qquad c^{(1)}_{2,4} = \texttt{0xe623bb72}$$
$$c^{(2)}_{2,0} = \texttt{0x7ad8818f}, \qquad c^{(2)}_{2,4} = \texttt{0x5c58a4a4}$$
$$c^{(3)}_{2,0} = \texttt{0x8438764a}, \qquad c^{(3)}_{2,4} = \texttt{0x1e38e2e7}$$
$$c^{(4)}_{2,0} = \texttt{0xbb6de032}, \qquad c^{(4)}_{2,4} = \texttt{0x78e38b9d}$$
$$c^{(5)}_{2,0} = \texttt{0xedb780c8}, \qquad c^{(5)}_{2,4} = \texttt{0x27586719}$$
$$c^{(6)}_{2,0} = \texttt{0xd9847356}, \qquad c^{(6)}_{2,4} = \texttt{0x36eda57f}$$
$$c^{(7)}_{2,0} = \texttt{0xa2c78434}, \qquad c^{(7)}_{2,4} = \texttt{0x703aace7}$$

B–3 $w = 4$

$$c^{(0)}_{3,0} = \texttt{0xb213afa5},\quad c^{(0)}_{3,4} = \texttt{0xe028c9bf}$$
$$c^{(1)}_{3,0} = \texttt{0xc84ebe95},\quad c^{(1)}_{3,4} = \texttt{0x44756f91}$$
$$c^{(2)}_{3,0} = \texttt{0x4e608a22},\quad c^{(2)}_{3,4} = \texttt{0x7e8fce32}$$
$$c^{(3)}_{3,0} = \texttt{0x56d858fe},\quad c^{(3)}_{3,4} = \texttt{0x956548be}$$
$$c^{(4)}_{3,0} = \texttt{0x343b138f},\quad c^{(4)}_{3,4} = \texttt{0xfe191be2}$$
$$c^{(5)}_{3,0} = \texttt{0xd0ec4e3d},\quad c^{(5)}_{3,4} = \texttt{0x3cb226e5}$$
$$c^{(6)}_{3,0} = \texttt{0x2ceb4882},\quad c^{(6)}_{3,4} = \texttt{0x5944a28e}$$
$$c^{(7)}_{3,0} = \texttt{0xb3ad2208},\quad c^{(7)}_{3,4} = \texttt{0xa1c4c355}$$

B–4 $w = 5$

$$c^{(0)}_{4,0} = \texttt{0xf0d2e9e3},\quad c^{(0)}_{4,4} = \texttt{0x5090d577}$$
$$c^{(1)}_{4,0} = \texttt{0xac11d7fa},\quad c^{(1)}_{4,4} = \texttt{0x2d1925ab}$$
$$c^{(2)}_{4,0} = \texttt{0x1bcb66f2},\quad c^{(2)}_{4,4} = \texttt{0xb46496ac}$$
$$c^{(3)}_{4,0} = \texttt{0x6f2d9bc9},\quad c^{(3)}_{4,4} = \texttt{0xd1925ab0}$$
$$c^{(4)}_{4,0} = \texttt{0x78602649},\quad c^{(4)}_{4,4} = \texttt{0x29131ab6}$$
$$c^{(5)}_{4,0} = \texttt{0x8edae952},\quad c^{(5)}_{4,4} = \texttt{0x0fc053c3}$$
$$c^{(6)}_{4,0} = \texttt{0x3b6ba548},\quad c^{(6)}_{4,4} = \texttt{0x3f014f0c}$$
$$c^{(7)}_{4,0} = \texttt{0xedae9520},\quad c^{(7)}_{4,4} = \texttt{0xfc053c31}$$

C Test Vectors

Let the message M be the 24 bits ASCII string "`abc`". Then the resultant message digest of each algorithm is as follows.

C–1 *Luffa*-224

The message digest of the message "`abc`" is

$$
\begin{aligned}
Z_{0,0} &= \texttt{0xf29311b8}, & Z_{0,1} &= \texttt{0x7e9e40de},\\
Z_{0,2} &= \texttt{0x7699be23}, & Z_{0,3} &= \texttt{0xfbeb5a47},\\
Z_{0,4} &= \texttt{0xcb16ea4f}, & Z_{0,5} &= \texttt{0x5556d47c},\\
Z_{0,6} &= \texttt{0xa40c12ad}.
\end{aligned}
$$

C–2 *Luffa*-256

The message digest of the message "`abc`" is

$$
\begin{aligned}
Z_{0,0} &= \texttt{0xf29311b8}, & Z_{0,1} &= \texttt{0x7e9e40de},\\
Z_{0,2} &= \texttt{0x7699be23}, & Z_{0,3} &= \texttt{0xfbeb5a47},\\
Z_{0,4} &= \texttt{0xcb16ea4f}, & Z_{0,5} &= \texttt{0x5556d47c},\\
Z_{0,6} &= \texttt{0xa40c12ad}, & Z_{0,7} &= \texttt{0x764a73bd}.
\end{aligned}
$$

C–3 *Luffa*-384

The message digest of the message "`abc`" is

$$
\begin{aligned}
Z_{0,0} &= \texttt{0x9a7abb79}, & Z_{0,1} &= \texttt{0x7a840e2d},\\
Z_{0,2} &= \texttt{0x423c34c9}, & Z_{0,3} &= \texttt{0x1f559f68},\\
Z_{0,4} &= \texttt{0x09bdb291}, & Z_{0,5} &= \texttt{0x6fb2e9ef},\\
Z_{0,6} &= \texttt{0xfec2fa0a}, & Z_{0,7} &= \texttt{0x7a69881b},\\
Z_{1,0} &= \texttt{0xe9872480}, & Z_{1,1} &= \texttt{0xc635d20d},\\
Z_{1,2} &= \texttt{0x2fd6e95d}, & Z_{1,3} &= \texttt{0x046601a7}.
\end{aligned}
$$

C–4 *Luffa*-512

The message digest of the message “`abc`” is

$$
\begin{aligned}
Z_{0,0} &= \texttt{0xf4024597}, & Z_{0,1} &= \texttt{0x3e80d79d},\\
Z_{0,2} &= \texttt{0x0f4b9b20}, & Z_{0,3} &= \texttt{0x2ddd4505},\\
Z_{0,4} &= \texttt{0xb81b8830}, & Z_{0,5} &= \texttt{0x501bea31},\\
Z_{0,6} &= \texttt{0x612b5817}, & Z_{0,7} &= \texttt{0xaae38792},\\
Z_{1,0} &= \texttt{0x1dcefd80}, & Z_{1,1} &= \texttt{0x8ca2c780},\\
Z_{1,2} &= \texttt{0x20aff593}, & Z_{1,3} &= \texttt{0x45d6f91f},\\
Z_{1,4} &= \texttt{0x0ee6b2ee}, & Z_{1,5} &= \texttt{0xe113f0cb},\\
Z_{1,6} &= \texttt{0xcf22b643}, & Z_{1,7} &= \texttt{0x81387e8a}.
\end{aligned}
$$

D Implementations of `SubCrumb`

D–1 For Intel Core2 Processors

The instructions are given by Table 4. At the first, the four words data a_0, a_1, a_2, a_3 are loaded to the registers `r0, r1, r2, r3` respectively. Then the resultant registers `r4, r1, r2, r3` provides the outputs of Sbox, namely, $x_0 =$ `r4`, $x_1 =$ `r1`, $x_2 =$ `r2`, $x_3 =$ `r3`.

Table 4: The instructions set for Intel Core2 processors

cycle			
1	`MOV r4 r0`	`OR r0 r1`	`XOR r2 r3`
2	`NOT r1`	`XOR r0 r3`	`AND r3 r4`
3	`XOR r1 r3`	`XOR r3 r2`	`AND r2 r0`
4	`NOT r0`	`XOR r2 r1`	`OR r1 r3`
5	`XOR r4 r1`	`XOR r3 r2`	`AND r2 r1`
6	`XOR r1 r0`		

E Implementations of Message Injection Function MI

The message injection function MI defined in Section 3.2 can be implemented only with XORings and multiplications by a fixed constant `0x02`.

E–1 $w = 3$

The matrix representation can be transformed as follows:

$$\begin{pmatrix} 3 & 2 & 2 & 1 \\ 2 & 3 & 2 & 2 \\ 2 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \oplus \begin{pmatrix} 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 \\ 2 & 2 & 2 & 0 \end{pmatrix} \oplus \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 4 \end{pmatrix}.$$

In other words, the message injection function MI for $w = 3$ can be also defined by the following equation:

$$X_j = H_j^{(i-1)} \oplus \left(\mathtt{0x02} \cdot \bigoplus_{j'=0}^{2} H_{j'}^{(i-1)} \right) \oplus \mathtt{0x02}^j \cdot M^{(i)}, \quad 0 \le j < 3,$$

Figure 8 shows an implementation image of MI for $w = 3$.

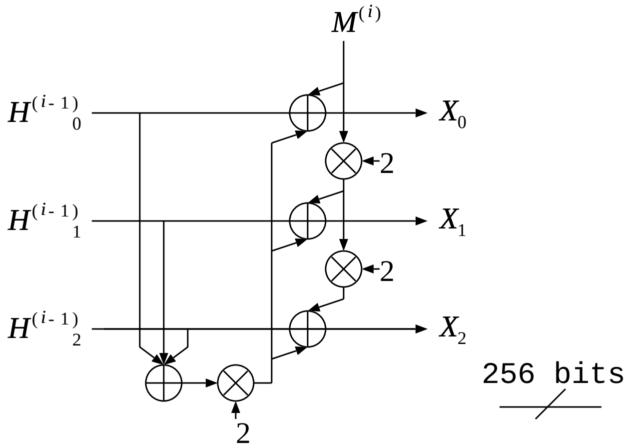

Figure 8: The message injection function ($w = 3$)

E–2 $w = 4$

The message injection function MI for $w = 4$ can be also defined by the following equations for $0 \le j < 4$:

$$\begin{aligned} \eta_j &= H_j^{(i-1)} \oplus \left(\texttt{0x02} \cdot \bigoplus_{j'=0}^{3} H_{j'}^{(i-1)} \right), \\ X_j &= \texttt{0x02} \cdot \eta_j \oplus \eta_{j-1 \bmod 4} \oplus \texttt{0x02}^j \cdot M^{(i)}. \end{aligned}$$

Figure 9 shows an implementation image of MI for $w = 4$.

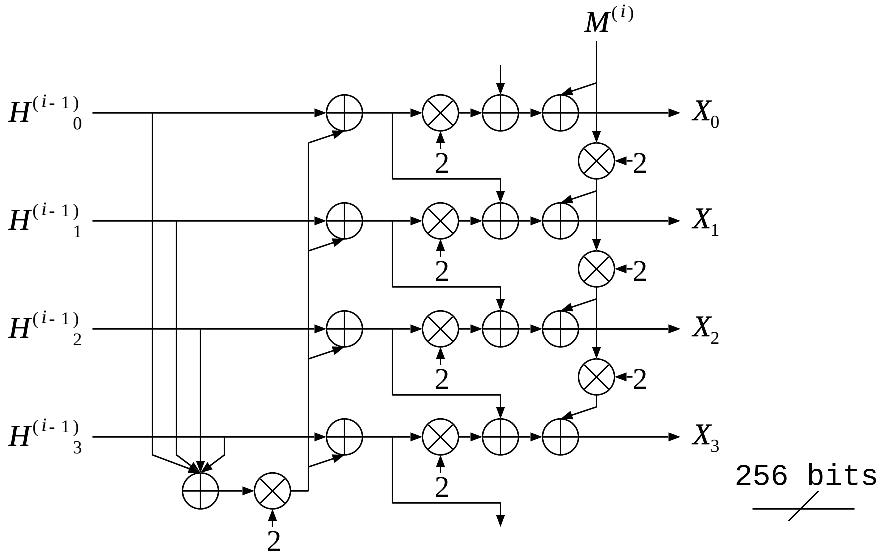

Figure 9: The message injection function ($w = 4$)

E–3 $w = 5$

The message injection function MI for $w = 5$ can be also defined by the following equations for $0 \leq j < 5$:

$$\begin{aligned}\eta_j &= H_j^{(i-1)} \oplus \left(\texttt{0x02} \cdot \bigoplus_{j'=0}^{4} H_{j'}^{(i-1)}\right), \\ \xi_j &= \texttt{0x02} \cdot \eta_j \oplus \eta_{j+1 \bmod 5}, \\ X_j &= \texttt{0x02} \cdot \xi_j \oplus \xi_{j-1 \bmod 5} \oplus \texttt{0x02}^j \cdot M^{(i)}.\end{aligned}$$

Figure 10 shows an implementation image of MI for $w = 5$.

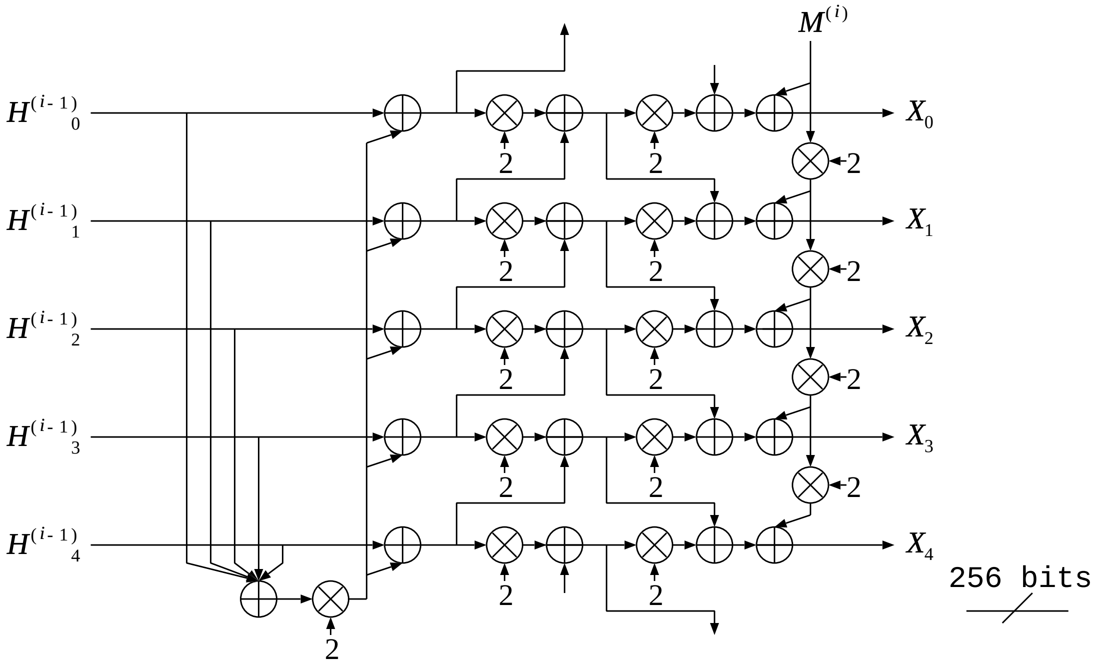

Figure 10: The message injection function ($w = 5$)